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## ABSTRACT

The scat tered fields from axisymmetric problems containing lossy dielectrics and an anisotropic media characterized by a 10 ss1css permeability tensor are found by the Hybrid Symmetric Finite Element (HSFEM) method. This method, recently applied to lossless ferrite objects, is applied to a lossy dielectric sphere. Extension of this method to scattering from cylindrical horns is discussed.

Keywords: Hybrid Symmetric Finite Element (HS-FEM) method, Lossy, Ferrite, Scattering, Radiation

#### 1. 1. Introduction

Development of hybrid methods that combine the benefits of the finite element method with those of integral equation techniques is allowing the analysis of more complicated objects. Specifically, it has recently been shown that these methods can be applied to bodies of revolution such that the resulting system matrix is complex and symmetric (Ref. 1). The resulting system is well conditioned and diagonally dominant, In addition, the computational boundary can be placed on the scattering object in a manner similar to integral equation solutions.

The advantage of the extra work in formulating a so-called exact boundary condition comes when the outer boundary is placed on the object. This minimizes the! large numb' of unknowns associated with the finite element basis functions, but allows their use inside inhomogeneous, lossy objects. Lossy dielectric objects are easily handled by inclusion of complex permittivities. Of even more interest, anisotropic media cau be handled without, the need for complicated Green's functions or difficult modal expansions. In particular, this method has been applied to ferrite materials (Ref. 2, 3). Ferrites are of interest, due to the study of the quasi-

optical ferrite circulator, for possible J 1'1 /NASA 1 Deep Space Network usc. '1 'he method has been extended so that the scattering geometry can be illuminated by a Gaussian incident field. The Faraday rotation through the device can be found and the influence of the finite edge effects determined.

### 2. II. RADIATION PROBLEMS

In order to study radiation from bodies of revolution it is necessary to modify the approach to include the sources that are internal to the computational boundary. Typically objects such as low gain antennas used in spacecraft communications have axisymmetric geometries. These antennas are often excited by cylindrical waveguide modes, see Figure 1. The implementation of this method for radiating bodies of revolution will be discussed.

The surface integral is used to truncate the finite element region using the hybrid symmetric formulation of Ref. 1, but also includes the effect of the modal expansion:

$$\begin{bmatrix} Z^{EE} & B^{EJ} & 0 & B^{EB} \\ B^{JE} & G^{JJ} & G^{JM} & 0 \\ 0 & G^{MJ} & G^{MM} & 0 \\ B^{BE} & 0 & 0 & K \end{bmatrix} \begin{pmatrix} C^{E} \\ C^{J} \\ C^{M} \\ b \end{pmatrix}$$

$$= \begin{pmatrix} a^{B} \\ V^{J} \\ V^{M} \\ a^{E} \end{pmatrix} \quad (1)$$

where  $B^{EB}$  and  $B^{BE}$  account for the interactions between modal and Finite Element regions.

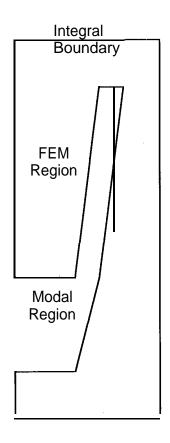
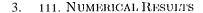
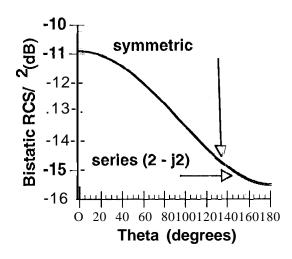


Figure 1: Cylindrical horn showing modal and FEM regions



Consider the example of plane wave scattering from a dielectric sphere where  $\epsilon_r = 2.0$  - j2.0, and the radius of the sphere is R = 1.59 mm. The finite element mesh consisted of 452 triangular elements giving a matrix order of 1,006 for the 71 =  $\pm$  1 modes. An outer boundary consisting of 19 triangular subdomain basis functions was placed on the sphere. Figures 1 and 2 show the computed results for the bistatic RCS for the lossy sphere.



**Figure** 2: *Plot of the bistatic*  $\phi \phi$  *RCS of a 10SSY dielectric Sphere,*  $\epsilon_T$ : **2.0**- j2.0, f= **30.0** *GHz.* 

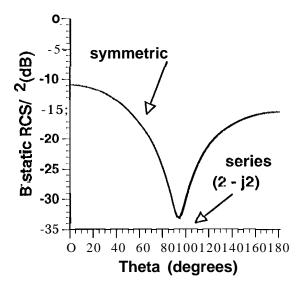


Figure 3: Plot of the bistatic  $\theta\theta$  RCS of a lossy dielectric sphere,  $\epsilon_r = 2.0$ -- j2.0, f= 30.0 GHz.

The sphere is excited by an incident plane wave from  $\theta = 180.0^{\circ}$  and  $\phi = 0.0^{\circ}$  requiring solution of only the  $n = \pm 1$  harmonics.

# 4. IV, CONCLUSIONS

The Hybrid Symmetric Finite Element was applied to lossy dielectric materials. The extension of this method to predict the radiation from cylindrical horns was discussed.

### 5. ACKNOWLEDGEMENT

This work was performed at the Jet Propulsion Laboratory, California Institute of Technology under a contract with the National Aeronautics and Space Administration. Support for this computing research was provided by the JPL Supercomputing Project on the CRAY computers at the Jet Propulsion laboratory and at Goddard Space Flight Center. The JJ 'J, Supercomputing Project is sponsored by the Jet 1'repulsion laboratory and the NASA Office of Space Sciences and Applications.

### References

[I] D. J. Hoppe, L. W. Epp, and J. F. Lee, "A hy-

- brid symmetric FEM/MOM formulation applied to scattering by inhomogeneous bodies of revolution," to appear in *IEEE Trans. Antennas Propagat.*, June 1994.
- [2] G.C.Chinn, 1., W. Epp, and G.M. Wilkins, "Determination of the eigenfrequencies of a ferrite filled cylindrical cavity resonator using the finite element method," submitted to IEEE Trans. on Microwave Theory and Tech.
- [3] 1. W. Epp, 1). J. Hoppe, and G. C. Chinn, "Scattering from ferrite bodies of revolution using a hybrid approach," submitted to IEEE Trans. Antennas Propagat.